JRAHS 2005 TRIAL HSC – MATHEMATICS (2 Unit)

Question 1. **[Start a New Page]** **Marks**

(a) Evaluate $\dfrac{8\pi}{2+\sqrt[3]{2}}$ as a decimal correct to 3 significant figures. **1**

(b) Find $\dfrac{d}{dx}[5x+\tan x]$. **2**

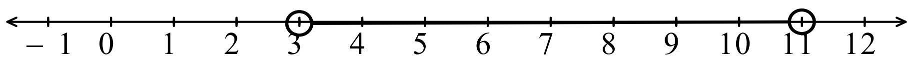
(c) A group of cards are labelled 0, 1, 2, … and 20. **2**
Find the probability that when a card is chosen, it is a prime number?

(d) Find $\int \sec 3x \tan 3x \, dx$, using the table of standard integrals. **2**

(e) Find the exact value for $\sec\dfrac{\pi}{6}$. **1**

(f) The number line graph represents the solutions to the inequality equation **2**

$$|x-a| \le b.$$



Find the values of a and b.

(g) Find integer p so that $(3-\sqrt{5})^2 = 14-\sqrt{p}$. **2**

Question 2. **[Start a New Page]**

(a) Differentiate with respect to x the following:

(i) $\dfrac{\sin x}{x+1}$. **2**

(ii) $\sqrt{1+e^{6x}}$. **2**

(iii) $x^3 \ln x$. **2**

(b) Find the domain for the function: $y = x + \ln(3 - x)$. **1**

(c) Solve for θ: $\tan\theta = 0\cdot 3$ (correct to two decimal places) for $0 < \theta < 2\pi$. **2**

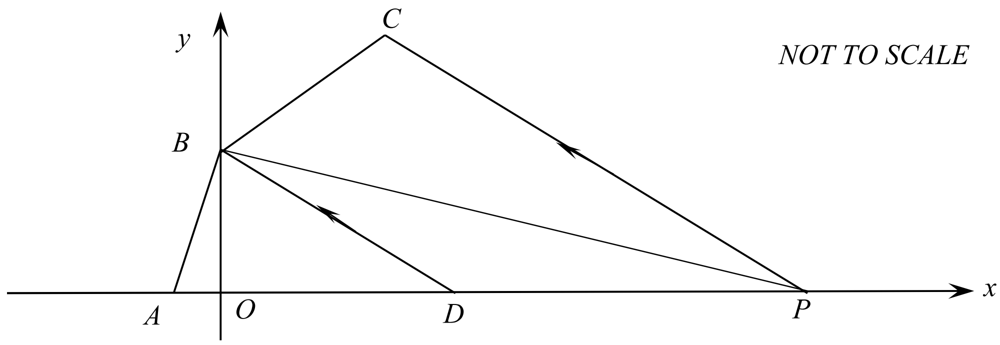
(d) Find a primitive function for $\dfrac{1}{3x}$. **1**

(e) Evaluate $\int_0^2 (e^{-x} + 1)\,dx$, to two decimal places. **2**

Question 3. **[Start a New Page]** **Marks**

(a) Find the equation of the normal to the curve $y = 2\cos x + 3$ at the point $(\frac{\pi}{2}, 3)$. **2**

(b) The diagram shows the coordinates of four points:
$A\ (-1, 0)$, $B(0, 2)$, $C(3, 5)$ and $D(8, 0)$.



(i) Find the gradient of *BD*. **1**

(ii) Find the equation of the line passing through *B* and *D*. **1**

(iii) Find the angle of inclination of the line *BD* (to nearest the degree). **1**

(iv) Given $CP \parallel BD$, show that the equation of *CP* is $x + 4y - 23 = 0$. **2**

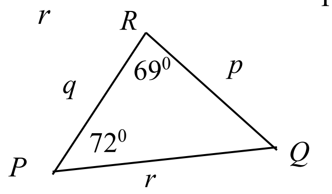
(v) Find the coordinates of point *P*, where the line *CP* intersects the *x*-axis. **1**

(vi) Find the perpendicular distance of point *C* from the line segment *BD*. **2**

(vii) Explain why the area of quadrilateral *ABCD* is the same as the area of triangle *ABP*. **2**

Question 4. **[Start a New Page]** **Marks**

(a) For ΔPQR, find $\dfrac{p}{r}$ correct to 3 decimal places. **2**

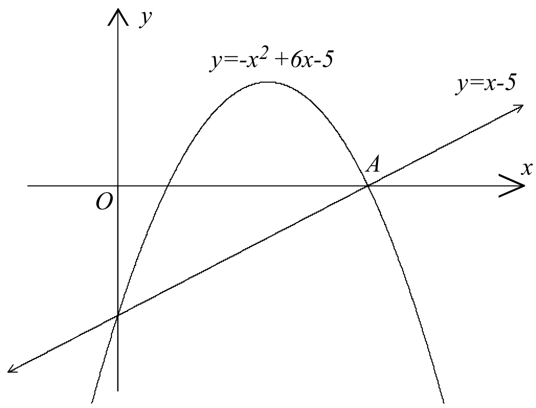


NOT TO SCALE

(b) The roots of the equation $x^2 - 2x - 5 = 0$, are $x = \alpha$ and $x = \beta$. **2**

Find the value of $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$.

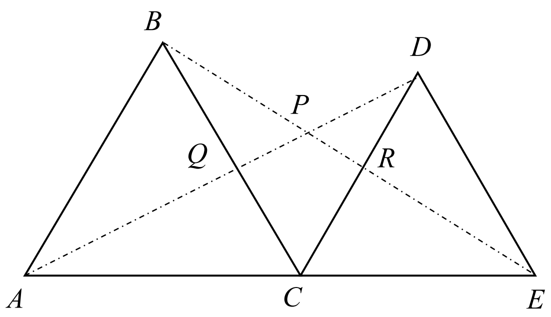
(c) The diagram shows the sketch of the parabola $y = 6x - x^2 - 5$ and a line $y = x - 5$.



(i) Find the x-coordinate of A. **1**

(ii) Find the area of the shaded region bounded by the line and the parabola. **2**

(d) Given the triangles ABC and CDE are different equilateral triangles. BE intersects AD at P, and A, C and E lie on the line segment AE.



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(i) *Copy* the diagram onto your writing booklet, and prove that $\Delta ACD \equiv \Delta ECB$. **3**

(ii) Show that $\angle APB = 60^0$. **2**

Question 5. **[Start a New Page]** **Marks**

(a) Simplify: $1+\cos^2 x+\cos^4 x+\ldots$ for $0<x<\dfrac{\pi}{2}$. **1**

(b) The first term of an Arithmetic series is a, the common difference is d and the n^{th} term is L.

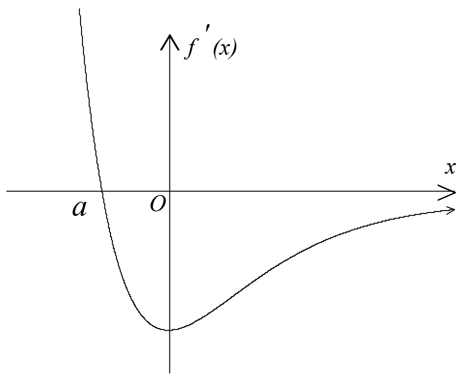
(i) Write down L, in terms of a, d and n. **1**

(ii) Show that the sum, S_n, of the first n terms can be expressed as **2**

$$S_n=\frac{(L+a)}{2}\left[1+\frac{L-a}{d}\right].$$

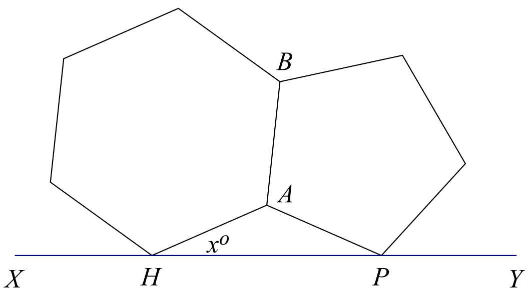
(iii) Hence, or otherwise, find: $5+8+11+\ldots+173$. **1**

(c) The sketch of the Gradient function $f'(x)$ is shown below. **3**



Sketch the graph of the function, $y=f(x)$, given $f(x)>2$ for $x>0$.

(d) The figure consists of a regular hexagon and a regular pentagon, with a common side AB. **4**



Given vertex H and P lie on a straight line $XHPY$ and $\angle PHA=x^0$.

Copy the diagram onto your writing booklet and find the value of x. Give reasons.

Question 6. **[Start a New Page]** **Marks**

(a) Solve for x: $\log_3(2x-5)=1$. **2**

(b) Differentiate $f(x)=\dfrac{3}{x}$ with respect to x, by first principles. **2**

(c) A particle is moving in a straight line with velocity $v=3e^{t}+6e^{-t}$.
It begins its motion at the Origin O, t is in seconds and v is in metres per second.

(i) What is its initial velocity? **1**

(ii) Is the particle ever at rest? Give reasons **1**

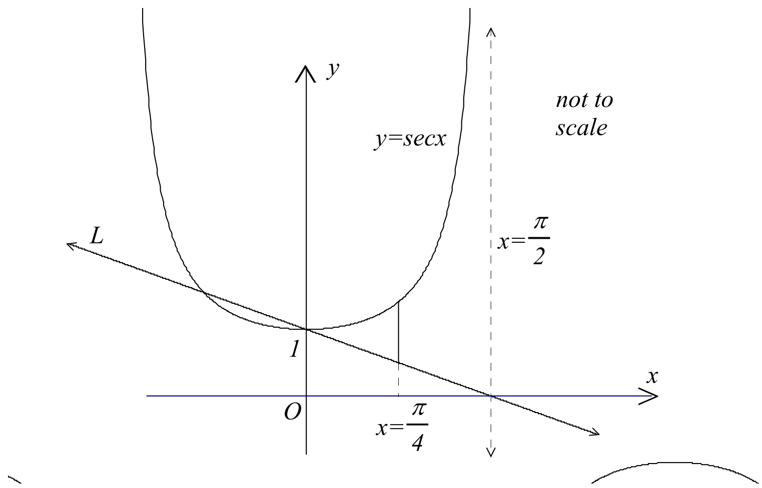
(iii) Find the displacement function, x, of the particle, at time t minutes. **2**

(iv) Find the time when the particle is at $x=10$. **2**

(d) Sketch the graph of the curve $y=3\sin 2x$ for the interval $0\le x\le\pi$. **2**

Question 7. **[Start a New Page]** **Marks**

(a) Given the sketch of the curve $y = \sec x$, for $0 \le x < \frac{\pi}{2}$ and the line L as shown.



(i) Show that the equation of the line, L, is $y = 1 - \frac{2}{\pi}x$. **1**

(ii) The area enclosed by the curve $y = \sec x$, the lines L and $x = \frac{\pi}{4}$, **3**
is rotated about the x-axis.
Find the exact volume for the solid of revolution.

(b) Use Simpson's rule, with three function values, to evaluate : $\int_0^4 \frac{3dx}{1+\sqrt{x}}$, **2**
(correct to two decimal places).

(c) Consider the function: $f(x) = x^3 - 3kx + 4$.

(i) Explain why $f(x)$ is an increasing function for all x when $k < 0$. **2**

(ii) Find the expression for each stationary point of $f(x)$, **2**
in terms of k, when $k > 0$.

(iii) Prove that $f(x)$ has 3 distinct real x-intercepts when $k^3 > 4$. **2**

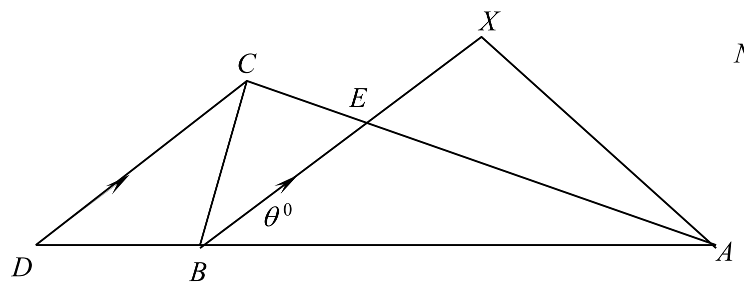
Question 8. **[Start a New Page]** **Marks**

(a) (i) Show that the locus of all points $P(x, y)$, which are equidistant from the origin O and to the line $y = 4$, is the parabola: **2**

$$x^2 = 16 - 8y.$$

(ii) Hence, find the coordinates of the vertex V. **1**

(b) In the diagram $BX \parallel DC$, XB bisects angle ABC and $AX \perp BX$ at X.
Let $\angle ABX = \theta^0$.



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Copy this diagram onto your writing booklet.

(i) State why $\angle BCD = \angle XBC$. **1**

(ii) Show that triangle BCD is isosceles. **2**

(iii) Hence, explain why $\dfrac{AE}{EC} = \dfrac{AB}{BC}$. **2**

(iv) If $\dfrac{BA}{BC} = 3$, and by using the cosine rule, or otherwise, show that E is the midpoint of BX. **4**

Question 9. **[Start a New Page]** **Marks**

(a) The mass of a substance X is M grams, at time t years.
It decays at an instantaneous rate proportional to its mass at time t years,
ie $\frac{dM}{dt} = -kM$, where k is the decay rate constant of proportionality.

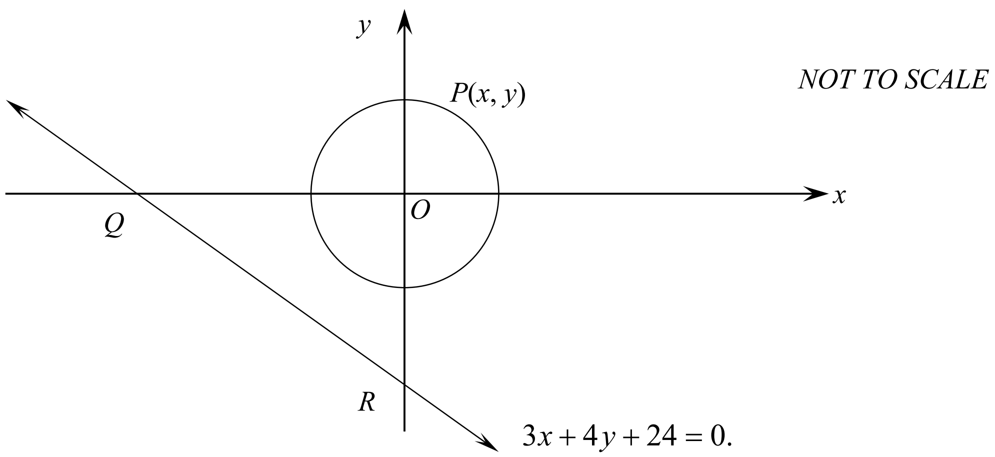
(i) Verify that $M = M_0 e^{-kt}$ satisfies the rate equation $\frac{dM}{dt} = -kM$, where M_0 is the initial mass of substance X. **1**

(ii) Hence, show that the time, T years, for half the mass of X to decay, is given by $T = \frac{\ln 2}{k}$. **1**

(iii) Find the decay rate constant, if the half-life of substance X is 3 466 years? **1**

(iv) Sketch the graph of $\frac{dM}{dt}$ against M. **1**

(b) Consider the circle: $x^2 + y^2 = 1$ and the line: $3x + 4y + 24 = 0$.
The points Q and R are the x and y-intercepts of the line $3x + 4y + 24 = 0$.
The point $P(x, y)$ lies on the circle as shown in the diagram.



(i) As P moves around the circle, show that the perpendicular distance, W, from the length QR to the circle, is given by: $W = 3x + 4y + 24$. **3**

(ii) Hence, or otherwise, find the least length of W. **5**

Question 10. **[Start a New Page]** **Marks**

(a) Mary visits the sock section of a shop that has 5 different pairs of socks individually arranged on a table.
She randomly selects socks one at a time.

(i) Explain why the probability that Mary does *not* have a matching pair of socks, after selecting the second sock, is $\frac{8}{9}$. **1**

(ii) Find the probability she does *not* have a matching pair of socks after selecting the third sock. **2**

(iii) What is the probability that, in the first 3 socks, Mary does have a matching pair? **1**

(b) Mr Howzat borrows $30 000 from a bank. Interest is to be calculated at 12% *pa*, compounded monthly, on the balance remaining over the term of the loan of 7 years.
Each year, at k regular intervals, (where $k = 1, 2, 3, \ldots$ or 12), Mr Howzat repays $$F$ for each instalment.

(i) Show that the amount owing, $$A_2$, after the second instalment is paid, is given by: **2**

$$A_2 = 30\,000\left[(1\cdot01)^{\frac{12}{k}}\right]^2 - F[1+(1\cdot01)^{\frac{12}{k}}].$$

(ii) Show that the amount of each instalment, $$F$, is given by: **3**

$$F = 30\,000 \times 1\cdot01^{84} \times \frac{[1\cdot01^{\frac{12}{k}} - 1]}{[1\cdot01^{84} - 1]}.$$

(iii) Calculate the value of each instalment if the instalments are made quarterly ($k = 4$). **1**

(iv) How much would Mr Howzat have saved over the term of the loan if he had chosen to make monthly rather than quarterly instalments? **2**

THE END

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JRAHS MATHEMATICS 2005

SOLUTIONS (2 UNIT)

TRAIL

MARKS

Q1 (a) $\frac{8\pi}{(2+\sqrt[3]{2})} = 7.7066\ldots$

$= 7.71$ (3 SF) ① — 1

(b) $\frac{d}{dx}[5x + \tan x]$

$= 5 + \sec^2 x$ ② — 1, 1

(c) $P(E = \text{prime}) = \frac{8}{21}$ ② — 1, 1

$\{2, 3, 5, 7, 11, 13, 17, 19\}$

(d) $\int \sec 3x \tan x \, dx$ ②

$= \frac{1}{3}\sec 3x + c$ — 1, 1

(e) $\sec\frac{\pi}{6} = \sec 30°$ ①

$\frac{1}{\sqrt{3}/2}$ $= \frac{2}{\sqrt{3}}$ — 1

(f) $a = \frac{3+11}{2} = 7$ ② — 1

$b = 7 - 3 = 4$ — 1

(g) $(3-\sqrt{5})^2 = 9 - 6\sqrt{5} + 5$

$= 14 - 6\sqrt{5}$ — 1

$\therefore -6\sqrt{5} = \sqrt{p}$

$p = 36 \times 5 = 180$ — 1

②

Q2 (a) (i) $y = \frac{\sin x}{x+1}$ ②

$\therefore \frac{dy}{dx} = \frac{\cos x \cdot (x+1) - 1 \cdot \sin x}{(x+1)^2}$ — 1, 1

$= \frac{(x+1)\cos x - \sin x}{(x+1)^2}$

(ii) $y = \sqrt{1+e^{6x}} = (1+e^{6x})^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{2}(1+e^{6x})^{-\frac{1}{2}} \times 6e^{6x}$ — 1, 1

$= \frac{3e^{6x}}{\sqrt{1+e^{6x}}}$ ②

(iii) $y = x^3 \ln x$ ②

$\frac{dy}{dx} = 3x^2 \cdot \ln x + x^3 \cdot \frac{1}{x}$ — 1

$= 3x^2 \ln x + x^2$ — 1

(b) for D_f: $3 - x > 0$ ①

$\therefore D_f = \{x : x < 3\}$ — 1

(c) $\tan\theta = 0.3$

$\theta = 0.2914\ldots$ or $\pi + 0.2914\ldots$

$\therefore \theta = 0.29$ or 3.43 (2dp) — 1, 1

②

(d) $\int \frac{1}{3x} dx = \frac{1}{3}\ln x (+c)$ — 1

or $= \frac{1}{3}\ln(3x)(+c)$

(e) $\int_0^2 e^{-x} + 1 \, dx$

$= \left[-e^{-x} + x\right]_0^2$ — 1

$= (-e^{-2} + 2) - (-e^0 + 0)$

$= -e^{-2} + 2 + 1$ — 1

$= 3 - e^{-2}$

2.8646...

Q3 (a) $y = 2\cos x + 3$

$\frac{dy}{dx} = -2\sin x$

at $x = \frac{\pi}{2}$, $m_T = -2\sin\frac{\pi}{2} = -2$

$\therefore m_N = +\frac{1}{2}$ — 1

Eqn. of normal $(\frac{\pi}{2}, 3)$

$y - 3 = \frac{1}{2}(x - \frac{\pi}{2})$ — 1

$y = \frac{1}{2}x + 3 - \frac{\pi}{4}$ / $2x - 4y + 12 - \pi = 0$

②

(b) (i) $m_{BD} = \frac{2-0}{0-8} = -\frac{1}{4}$ ① — 1

(ii) Eqn. of BD: $y = -\frac{1}{4}x + 2$ — 1

$B(0, 2)$ $[x + 4y - 8 = 0]$ ①

(iii) $m_{BD} = \tan\theta = -\frac{1}{4}$

$\therefore \angle\theta = 165°58'$

nearest degree $= 166°$ ① — 1

(iv) $m_{CP} = m_{BD} = -\frac{1}{4}$ ② — 1

Eqn. of CP: $y - 5 = -\frac{1}{4}(x - 3)$

$4y - 20 = -x + 3$ — 1

$\therefore x + 4y - 23 = 0$

(v) at P $y = 0$

$\therefore x + 0 - 23 = 0 \Rightarrow x = 23$

$\therefore P = (23, 0)$. — 1

vi) $\perp$ dist. $= \frac{|1 \times 3 + 4 \times 5 - 8|}{\sqrt{1^2 + 4^2}}$ — 1

$C = (3, 5)$

$x + 4y - 8 = 0$ ②

$= \frac{15}{\sqrt{17}}$ units — 1

(vii) Now $|ABCD| = |ABD| + |BDC|$

but $|BDP| = |BDC|$ (same base BD, same perp. height between parallel lines) — 1

$\therefore |ABCD| = |ABD| + |BDP|$ ②

$= |ABP|$ q.e.d.

OR $|APB| = \frac{1}{2} \times 24 \times 2 = 24\ u^2$

$|ABD| = \frac{1}{2} \times 9 \times 2 = 9\ u^2$ (common)

Q4 (a) $\frac{p}{\sin 72°} = \frac{r}{\sin 69°}$ ②

so $\frac{p}{r} = \frac{\sin 72°}{\sin 69°} = 1.018719436\ldots$

$= 1.019$ (3dp) — 1

(b) $x^2 - 2x - 5 = 0$ ②

$\frac{1}{\alpha} + \frac{1}{\beta} = \frac{\alpha + \beta}{\alpha\beta} = \frac{+2}{-5} = \frac{-2}{5}$ — 1, 1

(c) (i) $y = x - 5$

at A $y = 0$ $\therefore x = 5$ — 1

(ii) Area $= \int_0^5 (y_U - y_L)\, dx$

$= \int_0^5 6x - x^2 - 5 - (x - 5)\, dx$ — 1

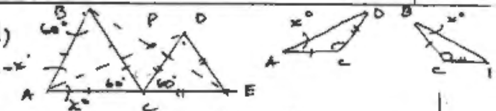
$= \int_0^5 (5x - x^2)\, dx$

$= \left[\frac{5}{2}x^2 - \frac{1}{3}x^3\right]_0^5$ ②

$= \frac{125}{2} - \frac{125}{3} - 0$

Area $= \frac{125}{6} = 20\frac{5}{6}\ u^2$. — 1

(d)



(i) In Δs ACD and ECD

S 1. $AC = CB$ (all sides equal in equil. ΔABC) — 1

A 2. $\angle ACD = \angle ECB = \angle BCD + 60°$ (all angles are 60° in equilateral Δs) — 1

S 3. $CD = CE$ ③

$\therefore \Delta ACD \equiv \Delta BCD$ (SAS) — 1

(ii) Let $\angle CAD = x°$

1. $\angle CBE = x°$ (corresp. angles in cong. triangles are equal)

2. $\angle BAP = 60° - x°$

3. $\angle APB + 60° - x° + 60° + x° = 180°$ (angles sum of ΔAPB is 180°) — 1

② $\therefore \angle APB = 60°$

* $BD = \sqrt{68}$

and $|BDC| = \frac{1}{2} \times \sqrt{68} \times \frac{15}{\sqrt{17}} = 15\ u^2$

$|BDP| = \frac{1}{2} \times \sqrt{68} \times \frac{15}{\sqrt{17}} = 15$

$\therefore |ABCD| = 9 + 15$

$= 24$

$\therefore |ABCD| = |ABP| = 24\ u^2$

6

a) $\log_3(2x-5) = 1$ (2)

$\therefore 2x - 5 = 3^1 = 3$ 1

$\therefore x = 4$ 1

b) $f'(x) = \lim_{h\to 0} \dfrac{\left(\frac{3}{x+h} - \frac{3}{x}\right)}{h}$ 1

$= \lim_{h\to 0} \left(\dfrac{3x - 3(x+h)}{hx(x+h)}\right)$ (2)

$= \lim_{h\to 0} \dfrac{-3h}{hx(x+h)}$ $h \neq 0$ and $h \to 0$

$= \lim_{h\to 0} \dfrac{-3}{x(x+h)}$ 1

$f'(x) = \dfrac{-3}{x(x+0)} = \dfrac{-3}{x^2}$

c) $V = 3e^t + 6e^{-t}$ $t=0$, $x=0$

initial velocity $= 3+6 = 9$ m/s 1 (1)

$e^t, e^{-t} > 0$ for all $t \geq 0$

$V = 3e^t + 6e^{-t} > 0$ (1) 1

particle never at rest. as $V \neq 0$

) $x = \int (3e^t + 6e^{-t})\,dt$

$x = 3e^t - 6e^{-t} + c$ 1

when $t = 0$ $x = 0$

$\therefore 0 = 3 - 6 + c$ (2)

$\therefore c = 3$ 1

so $x = 3e^t - 6e^{-t} + 3$

$\therefore x = 3e^t - 6e^{-t} + 3 = 10$

$3e^t - 6e^{-t} - 7 = 0$

$3e^{2t} - 6 - 7e^t = 0$ 1

$u = e^t$

$3u^2 - 7u - 6 = 0$

$(3u+2)(u-3) = 0$

$\therefore u = -\frac{2}{3}$ or $u = 3$

i.e. $e^t = -\frac{2}{3}$ or $e^t = 3$

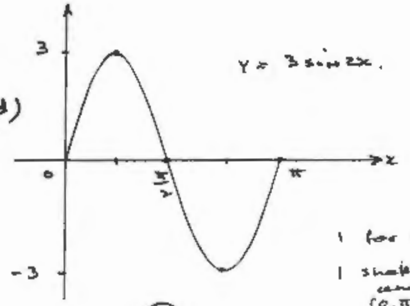
but $e^t > 0$ or $t = \ln 3$ 1

$\therefore$ no soln possible

$\therefore$ the time to be at $x = 10$

(2) is $\ln 3$ seconds

(d)



1 for Amp.

1 shape and $[0, \pi]$

(2)

Q5.

(a) $r = \cos^2 x$ $0 < x < \frac{\pi}{2}$ ($0 < r < 1$)

$\therefore S = \dfrac{a}{1-r} = \dfrac{1}{1-\cos^2 x}$ (1)

$S = \dfrac{1}{\sin^2 x} = \operatorname{cosec}^2 x$ 1

(b) (i) $T_n = L = a + (n-1)d$ (1) 1

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$ // $\frac{n}{2}[a + L]$*

but as $L = a + (n-1)d$

$(n-1) = \dfrac{L-a}{d}$ 1

$n = \dfrac{L-a}{d} + 1$

so $S_n = \frac{1}{2}\left(\frac{L-a}{d} + 1\right)\left[2a + \left(\frac{L-a}{d}\right)\times d\right]$

$= \frac{1}{2}\left(1 + \frac{L-a}{d}\right)[2a + L - a]$ 1

$\therefore S_n = \frac{1}{2}(a+L)\left(1 + \frac{L-a}{d}\right)$ (2)

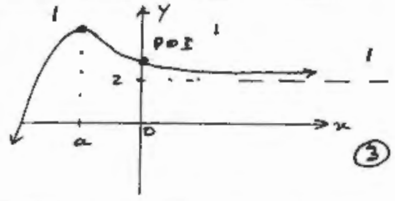
(iii) Hence: (1)

$S = \frac{1}{2}(5 + 173)\left(1 + \frac{173-5}{3}\right)$

$= \frac{1}{2} \times 178 \times (1 + 56)$

Sum $= 5073$. 1

c)



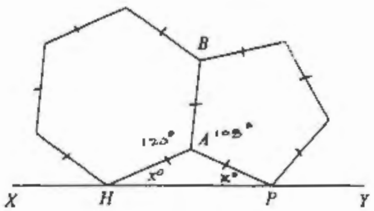
(3)

1 max TP

1 POI x=0

1 HA y=2

1 shape of f(x)



d) 1. $\angle BAH = \frac{(6-2)\times 180°}{6} = 120°$ 1

(all angles of reg. hexagon 120°)

2. $\angle BAP = \frac{(5-2)\times 180°}{5} = 108°$

(all angles of reg. pentagon 108°)

3. $\angle HAP + 120° + 108° = 360°$

(angle sum at pt. A is 360°) 1

$\therefore \angle HAP = 132°$

4. $AH = AP = AB$ (all sides of reg. hexagon + pentagon are equal)

5. $\angle APH = \angle AHP = x°$ (equal angles opposite equal sides) 1

6. $x + x + 132 = 180$ (angle sum of $\triangle AHP = 180°$)

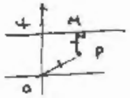
$2x = 48$

$x = 24$ 1

(4)

Q8

a) (i) $OP = PM$



$\sqrt{x^2+y^2} = |4-y|$

$x^2+y^2 = (4-y)^2$ 1

$x^2+y^2 = 16-8y+y^2$ 1

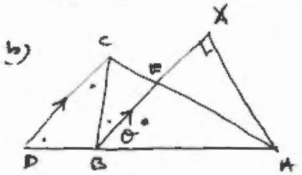
(2) $x^2 = 16-8y$

ii) $x^2 = 16-8y$

$x^2 = 4(-2)(y-2)$

$\therefore V = (0,2)$ (1) 1

b)



(i) $\angle BCD = \angle XBC$

Alternate angles equal as $DC \parallel BX$ (1) 1

ii) 1. $\angle CBX = \theta^\circ$ (Data) 1

2. $\angle CDB = \theta^\circ$ (corresponding angles equal as $DC \parallel BX$) 1

3. $\therefore \angle CDB = \angle DCB = \theta^\circ$

$\therefore \triangle BCD$ is isosceles (2)

(two angles equal)

iii) $\dfrac{AE}{EC} = \dfrac{AB}{BD}$ 1

but $DB = CB$ (equal sides opposite equal angles) 1

$\therefore \dfrac{AE}{EC} = \dfrac{AB}{CB}$ qed.

(2)

METHOD 1.

(iv) Let $BC = a = BD$

$\therefore BA = 3a$

and so $AE = 3EC$ part (iii) 1

$AE^2 = 9EC^2$

i.e. $BE^2 + 9a^2 - 6aBE\cos\theta$

$= 9[a^2 + BE^2 - 2aBE.\cos\theta]$ 1

$BE^2 + 9a^2 - 6aBE\cos\theta = 9a^2 + 9BE^2 - 18aBE\cos\theta$

$\therefore 8BE^2 = 12aBE\cos\theta$

$2BE = 3a\cos\theta$ 1

but $\cos\theta = \dfrac{BX}{BA} = \dfrac{BX}{3a}$ (3)

$\therefore 2BE = BX$

$\Rightarrow$ E is the midpt of BX

Q7. (a)

(i) $m_L = \dfrac{1-0}{0-\frac{\pi}{2}} = -\dfrac{2}{\pi}$ 1

$(0,1)$

$(\frac{\pi}{2}, 0)$

$\therefore$ Equ. of L: $y = -\frac{2}{\pi}x + 1$.

(ii) Volume $= \pi\int_0^{\pi/4} \sec^2 x\,dx - \pi\int_0^{\pi/4} (1-\frac{2}{\pi}x)^2\,dx$

$= \pi\int_0^{\pi/4} \sec^2 x - (1-\frac{2}{\pi}x)^2\,dx$

or $= \pi\int_0^{\pi/4} \sec^2 x - 1 + \frac{4}{\pi}x - \frac{4}{\pi^2}x^2\,dx$

$= \pi[\tan x + \frac{\pi}{6}(1-\frac{2}{\pi}x)^3]_0^{\pi/4}$ 1

or $= \pi[\tan x - x + \frac{2}{\pi}x^2 - \frac{4}{3\pi^2}x^3]_0^{\pi/4}$

$= \pi[(1+\frac{\pi}{6}(1-\frac{1}{2})^3) - (0+\frac{\pi}{6})]$ 1

$= \pi[1 + \frac{\pi}{48} - \frac{\pi}{6}]$ (3)

Vol $= \pi[1 - \frac{7\pi}{48}]$ u³ 1

(b) $I = \int_0^4 \dfrac{3\,dx}{1+\sqrt{x}}$

x	0	2	4
$\frac{3}{1+\sqrt{x}}$	3	$\frac{3}{1+\sqrt{2}}$ (1.2426...)	1

$\therefore I \doteqdot \frac{2}{3}[3 + 1 + 4\times\frac{3}{1+\sqrt{2}}]$ 1

$= \frac{2}{3}\times 8.97056\ldots$

$= 5.980375\ldots$ (2)

$\therefore I = 5.98$ (2 dp) 1

(c) (i) $f'(x) = 3x^2 - 3k$

If $k<0$ then $-3k>0$, $3x^2 \geq 0$ 1

so $f'(x) = 3x^2 - 3k > 0$

$\therefore f(x)$ is increasing $\forall x$. 1

(2)

(c) (ii)

For S.Ps to occur $f'(x) = 0$

$\therefore 3x^2 - 3k = 0$

$x^2 = k$

$k>0$ $x = \sqrt{k}$ or $-\sqrt{k}$

so $f(\sqrt{k}) = (\sqrt{k})^3 - 3k\sqrt{k} + 4$

$= k\sqrt{k} - 3k\sqrt{k} + 4$

$= 4 - 2k\sqrt{k}$

and $f(-\sqrt{k}) = (-\sqrt{k})^3 - 3k(-\sqrt{k}) + 4$

$= -k\sqrt{k} + 3k\sqrt{k} + 4$

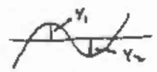
$= 4 + 2k\sqrt{k}$

$\therefore$ SPs are $(\sqrt{k}, 4-2k\sqrt{k})$ 1

(2) $(-\sqrt{k}, 4+2k\sqrt{k})$ 1

(iii) Since a cubic

For 3 distinct real roots



$Y_1 Y_2 < 0$

so $(4-2k\sqrt{k})(4+2k\sqrt{k}) < 0$ 1

$16 - 4k^2 k < 0$

$16 - 4k^3 < 0$

(2) $-4k^3 < -16$ 1

$\therefore k^3 > 4$

OR as $k>0$

$4 + 2k\sqrt{k} > 0$

but $4 - 2k\sqrt{k} < 0$ for 3 roots

i.e. $2k\sqrt{k} > 4$

$k\sqrt{k} > 2$

$\therefore k^3 > 4$

Q10.

(a) (i) METHOD 1. Not match.

$P(E) = 10 \times \frac{1}{10} \times \frac{10-2}{10-1} = \frac{8}{9}$

METHOD 2

1st → 2nd: $\frac{1}{9}$ M, $\frac{8}{9}$ $\bar{M}$

$P(E) = 8/9$ (1)

(ii) 2nd: $\frac{1}{9}$ M, $\frac{8}{9}$ $\bar{M}$; 3rd: $\frac{2}{8}$ M, $\frac{6}{8}$ $\bar{M}$

P(E = no match after 3 socks) (2)

$= \frac{8}{9} \times \frac{6}{8} = \frac{2}{3}$

(iii) P(E = a matching pair in 3)

$= 1 - P(\text{No match})$

$= 1 - \frac{2}{3}$

$= \frac{1}{3}$ (1)

or $P(MM) + P(M\bar{M}M) + P(\bar{M}MM)$

$5 \times [\frac{1}{10} \times \frac{1}{9} + \frac{1}{10} \times \frac{8}{9} \times \frac{1}{8} + \frac{8}{10} \times \frac{2}{8} \times \frac{1}{9}] = \frac{1}{3}$

(b) (i) r = 1% p month (2)

interval k gives $\frac{12}{k}$

$\therefore n = 7k$

Amount owing at end of 1st \$F

$A_1 = 30\,000 \times 1.01^{12/k} - F$

Amt. owing at end of 2nd \$F

$A_2 = A_1 \times 1.01^{12/k} - F$

$= 30\,000 \times 1.01^{2 \times 12/k} - F(1 + 1.01^{12/k})$

$A_2 = 30\,000 \times [1.01^{\frac{12}{k}}]^2 - F[1 + 1.01^{12/k}]$

Amt owing at end of n^{th} \$F

$A_n = 30\,000(1.01^{12/k})^n - F(1 + 1.01^{12/k} + 1.01^{24/k} + \ldots + 1.01^{(n-1)\frac{12}{k}}]$

but when $\frac{12}{k} \times n = 84$ or $n = 7k$

$A_{7k} = 0$ debt paid

Q10(b)(ii) (3)

$0 = 30\,000 \times 1.01^{84} - F[1 + 1.01 + \ldots + 1.01^{(7k-1)12/k}]$

$0 = 30\,000 \times 1.01^{84} - F \times a\frac{(r^n - 1)}{r - 1}$, $n = 7k$

i.e. $F \times \left[\frac{(1.01^{12/k})^n - 1}{1.01^{12/k} - 1}\right] = 30\,000 \times 1.01^{84}$

i.e. $F \times \left[\frac{1.01^{84} - 1}{1.01^{12/k} - 1}\right] = 30\,000 \times 1.01^{84}$

$\therefore F = \frac{30\,000 \times 1.01^{84}(1.01^{12/k} - 1)}{(1.01^{84} - 1)}$

(iii) $k = 4$ $F_4 = 1604.69$

$\frac{12}{k} = 3$ (1)

∴ 3 mths

(iv) $k = 12$, $\frac{12}{k} = 1$

$F_{12} = 529.58$

Save $= 28F_4 - 84F_{12}$

$= \$446.33$ (2)

Q9.

(a) (i) $\frac{dM}{dt} = -kM$

LHS: $\frac{d}{dt}M_0e^{-kt} = -kM_0e^{-kt}$

RHS: $-kM = -kM_0e^{-kt}$

∴ LHS = RHS

∴ $M = M_0e^{-kt}$ satisfies $\frac{dM}{dt} = -kM$.

(ii) $t = T$ $M = \frac{1}{2}M_0$ (2)

$\therefore \frac{1}{2}M_0 = M_0e^{-kT}$

$\frac{1}{2} = e^{-kT}$

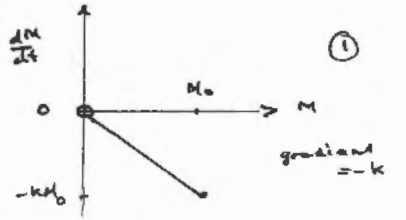
$\ln\frac{1}{2} = -kT$

$T = \frac{\ln\frac{1}{2}}{-k} = \frac{-\ln 2}{-k} = \frac{\ln 2}{k}$

(iii) $3466 = \frac{\ln 2}{k}$ (1)

$k = \frac{\ln 2}{3466} = 2 \times 10^{-4}$ (1SF)

(iv) (1)



negative gradient line segment

(b) (i) $W = \frac{|3x + 4y + 24|}{\sqrt{3^2 + 4^2}}$

$= \frac{|3x + 4y + 24|}{5}$

but as P on the RHS of the line $3x + 4y + 24 > 0$

$\therefore W = \frac{3x + 4y + 24}{5}$ qed. (2)

Q9(b)(ii) METHOD 1.

P(x,y) lies on $x^2 + y^2 = 1$

$\therefore y = \pm\sqrt{1 - x^2}$

but shortest distance will occur on lower semicircle

$\therefore y = -\sqrt{1 - x^2}$

$\therefore W = \frac{1}{5}(3x - 4\sqrt{1 - x^2} + 24)$

$\frac{dW}{dx} = \frac{1}{5}[3 - 4 \times \frac{1}{2}(1 - x^2)^{-\frac{1}{2}} \times -2x]$

$= \frac{1}{5}[3 + \frac{4x}{\sqrt{1 - x^2}}]$

For possible max/min. values of W to occur $\frac{dW}{dx} = 0$

$\therefore 3 + \frac{4x}{\sqrt{1 - x^2}} = 0$

so $4x = -3\sqrt{1 - x^2}$

so $x < 0$ for soln

$\therefore 16x^2 = 9(1 - x^2)$

$16x^2 = 9 - 9x^2$

$25x^2 = 9$

$x^2 = \frac{9}{25}$

$x = \pm\frac{3}{5}$

but $x < 0$ $x = -\frac{3}{5}$, $y = -\frac{4}{5}$

TEST at $x = -\frac{3}{5}$

x	-0.7	$-\frac{3}{5}$	$-\frac{1}{2}$
W'	-0.184	0	0.138
	⊖	—	⊕

($W' = \frac{3 + \frac{4x}{\sqrt{\ }}}{5}$; $-0.7 \le x \le -\frac{1}{2}$)

Since W is cont + diff'ble over and $\frac{dW}{dx}$ changes sign (− 0 +)

∴ a relative min. T.P at $x = -\frac{3}{5}$

Since no other TPs in $-1 \le x \le 0$

∴ (abs) min. length is

(5) $W = \frac{1}{5}[3(-\frac{3}{5}) + 4(-\frac{4}{5}) + 24]$

$= 3.8$ units